If a prime number 'p' divides a square number 'a^2', then will it even divide 'a'?

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In the previous segment, we saw **What irrational numbers are and why they are needed**. In this segment let us prove if '*p*' is a prime number such that '*p*' divides square of '*a*', then '*p*' divides '*a*'.

How do we prove If '*p*' is a prime number such that '*p*' divides square of '*a*', then '*p*' divides '*a*'?

Consider a positive integer a. Let the prime factors of a be $f_1, f_2, f_3 \dots f_n$ which are not necessarily distinct.

Thus, $a = f_1 \times f_2 \times f_3 \times \dots \times f_n$

$$\therefore a^2 = (f_1 \times f_2 \times f_3 \times \dots \times f_n) \times (f_1 \times f_2 \times f_3 \times \dots \times f_n)$$

$$\therefore a^2 = (f_1 \times f_1) \times (f_2 \times f_2) \times (f_3 \times f_3) \times \dots \times (f_n \times f_n)$$

It is given that the prime number $p\ divides\ a^2. This\ means\ a^2\ is\ divisible\ by\ p. Thus, (f_1 \times f_1) \times (f_2 \times f_2) \times (f_3 \times f_3) \times \dots \times (f_n \times f_n)\ is\ divisible\ by\ p.$

Therefore $\frac{p}{a}$ must be equal to any one of the prime factors among $f_1, f_2, f_3, \ldots f_n$.

But $a = f_1 \times f_2 \times f_3 \times \ldots \times f_n$. So, p is also one of the prime factors of a and will thus divide the number a too.

Hence, if p is a prime number such that p divides a^2, then p divides a where a is a positive integer.

Q. If 30276 is divisible by 29, will 174 be divisible by 29?

Solution:

$174^2 = 30276$

if p is a prime number such that p divides a^2, then p divides a where a is a positive integer.

So, since 30276 is divisible by 29, 174 should also be divisible by 29.

Summary

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What's next?

In the next segment of Class 10 Maths, we will look at the **Proof of existence of irrational numbers.**